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On the Nature of Necessity: Later Wittgenstein

I

At the beginning I would like to mention that later Wittgenstein did not really want to distinguish mathematical necessity from logical necessity or logical/mathematical necessity from grammatical/linguistic necessity. On the contrary, it seems that he was very eager to show that they go hand in hand, and that a clear parallel between them can be drawn. This is important, as we usually think of necessity as belonging exclusively to a priori propositions; and Wittgenstein has made necessity divorce its epistemic companion a priority in his later writings. However, I have also used the term 'necessity' accordingly in this paper. The context will determine whether the necessity I am talking about is logical, mathematical, or grammatical.

The notion of necessity has eluded philosophers for ages and Wittgenstein was preoccupied with the nature and problems centering round the notion of necessity throughout his career. This paper intends to offer a reading of his remarks on necessity as found in Lectures on the Foundations of Mathematics (LFM), Remarks on the Foundations of Mathematics (RFM) and Philosophical Investigations (PI). The trouble with reading the remarks of later Wittgenstein lies mainly in his opposition to theorization. Moreover, his interlocutor often puzzles its readers in making sense of his own views in contrast with his opponents. His remarks on this subject have invited several opposite interpretations and have conferred a special enigma to his views on necessity.

In this paper, I will explore the various arguments by which Wittgenstein attempted to refute the misconceptions that are conventionally intertwined with the notion of necessity. This paper has been divided into two main sections. In the first section, we discuss how Wittgenstein refutes the misconceptions involved in the notion of necessity, be it logical, or mathematical or grammatical. The approach of this section is purely negative, as it shows what necessity is not. As Wittgenstein was opposed to any sort of theorizing, it is very difficult to pinpoint the positive account of necessity in the philosophy of later Wittgenstein. However, an attempt has been made in the final section to find out the positive implications, if any, of his earlier criticisms of the conventional thinking about necessity.

At the very beginning of Remarks on the Foundations of Mathematics, we find Wittgenstein clearing the misconceptions usually associated with the notion of necessity. He repeatedly points out several temptations, which we succumb to, while explaining the use of necessity in our ordinary language. In Philosophical Investigations, he elucidates the problem of necessity by giving an example of a student who has been asked to add 2. He goes alright till 998, but from 1000 he goes on like this: 1004, 1008, 1012. Obviously, he has not done what the teacher meant him to do i. e., he has not followed the rule. And the right way of following instructions must be in this way: 1002, 1004, 1006, 1008. But in what exactly did my meaning him to continue the series with 1002, 1004 consist? In Wittgenstein's own words:

> How do I know that in working out the series 'add 2', I must write 2004, 2006 and not 2004, 2008? (The question how do I know that this color is red? is similar.) But you surely know for example that you must always write the same sequence...[1]

Wherefrom comes this surety? Wherein lies the source of this strange inexorability of logical must? Why is it that one cannot move according to his own interpretation? There are various philosophical positions answering to these questions:

> A: The way the formula is meant determines which steps are to be taken. It may be called the psychological or subjective account of necessity.
> B: The steps are determined by the formula. It may be termed as the functionalistic account of necessity.
> C: The steps are determined by the discoverable logical or mathematical reality. It may be termed as the Platonic account of necessity.
> D: The steps are determined by the conventions of our using language. It might be called the sociological account of necessity.

I will elaborate each of these in accordance with the views of later Wittgenstein.

A: The Psychological account of necessity: The source of this necessity lies in our meaning it. George Boole, the famous logician, subscribed

[1] Wittgenstein L. Remarks on the Foundations of Mathematics. – Oxford: Basil Blackwell, 1967, I.4.

to this view. For him, 'the laws of logic describe how human beings by and large think, their basic mental operations and are determined by the nature of the human mind.' In elucidating the position of this adversary, Wittgenstein says:

My experience shows that there is a connection between thinking of the formula and actually continuing the series. The implication is: if one really understands what the speaker means by saying '2' and 'add', he cannot but draw that particular conclusion. That is, the conclusion must follow from the premises[2].

Now following RFM and PI, I would like to distinguish among variants of experiences, which are usually attached to our actual continuation of series.

I: My future steps are already taken in my mind: Here the picture is of a mind, which contains all the future steps. It presupposes that when I ask my pupil to add 2; I, in effect, perform an act, a very special queer act of meaning, which determines an infinite number of future steps.

II: A segment of a series intimates or suggests to me how to continue the series. He elaborates:

> One could however imagine that someone multiplied, multiplied correctly, with such feelings, kept on saying, "I do not know how suddenly the rule intimates this to me and that we reply:" Of course; for you are going ahead perfectly in accordance with the rule[3].

Wittgenstein gives other variants of experiences as well – he talks about an image (PI, 156) a feeling, (PI, 184) an act or way of meaning, (PI, 188, 190) intuition, (PI, 186) an intention (PI, 197, 337). I will not elaborate upon these. It is now clear how the psychologist or subjectivist gives an account of necessity.

Wittgenstein opposed to such account of necessity. He argues:

> It appears that this rule (say add 2) as it was meant, foreshadowed all the transitions which were to be made according to it. But the assumption of a shadow of transition does not get us any further, because it does not bridge the gulf between it and real transition. If the

[2] Wittgenstein L. Philosophical Investigations. – Oxford: Basil Blackwell, 1953, §118.
[3] Wittgenstein L. Remarks on the Foundations of Mathematics. IV, 60, p. 421.

> mere words of the rule could not anticipate a future transition, no more could any mental act accompanying these words[4].

Thus it is a misconception to think of meaning as a queer mysterious act which determines all the right steps (say 1002 after 1000) when the speaker himself has never actually thought of this particular number of the series.

Regarding the feeling of intimation, image or intuition, Wittgenstein is of the opinion that someone might have some special feeling or some intimation that he will have to go on in a certain way and is continuing in that way, is not actually following a rule. It is also highly probable that someone is being intimated that he is not following a rule, while he is actually following the rule and continuing the series. (RFM, VII, 51, p. 416).

Wittgenstein goes on arguing that if we believe that 'the line intimates this to me as a rule – always the same', then private intuition alone / remains to be the only court of appeal for what I have to do; whereas it is, at the same time, evident that following a rule is not a private affair. (RFM, p. 419, VII, 857).

As we will see later, Wittgenstein attaches this notion of rule – following or necessity with that of practice. If it is a practice, then seeming to follow a rule is not following a rule. He confirms it and comments: 'the model seems to intimate to him how he has to go. But it is not a rule'. (RFM, VII, 49).

Hence one's own experience can never account for the necessity involved in following a rule and continuing a series.

But the question still persists: How are we to account for the meaning of the statement that the teacher meant the pupil to write 1002 after 1000? Wittgenstein answers:

> When you said "I already knew, at the time . . . that meant something like" . . if I had then been asked what number should be written after 1000. I should have replied 1002[5].

It is evident to Wittgenstein that, continuing the series in proper way, to follow the rule correctly, is an ability to react in a certain manner under certain circumstances. It is a technique, an ability to respond to a particular situation.

[4] Wittgenstein L. The Blue and Brown Books. – Oxford: Basil Blackwell, 8, 142-143.
[5] Wittgenstein L. Philosophical Investigations, §187

B: The Functionalistic account of necessity: The steps are determined by the formula (PI, 189).

We usually believe that rules are so fixed that they are like mathematical functions in the sense that they determine values for any indefinite number of arguments and applications. In fact, in the transitional period, Wittgenstein himself used to believe in such conception of a rule determining the steps ahead of time.

In his later years, he felt that the conception of the meaning of a sentence as being fixed or determined by a set of rules is totally mistaken. The main reason: he thinks that the word 'determine' has a variety of meanings. Floyd (1996) has elucidated various senses of the word 'determine'. The meaning of 'determine' is not itself determined. He mentions about the special confusion generated from not distinguishing between the different senses of 'determine'.

'The steps are determined by the algebraic formula' may mean either that people normally act in the same way in this connection, or it states about the operations of mathematics. As we find in his Lectures:

> Does this formula '$Y = X^2$' determine what is to happen at the 100^{th} step?
>
> This may mean: Is there any rule about it? Or do most people after being taught to square numbers upto 100, do so-and-so when they get to 100?" (which) is a completely different question. 'The former is about the operations of Mathematics, but the latter is about people's behaviour'[6].

When we confuse these two senses, we conceive that the rule determines the correct application of it in infinite number of instances. But 'this is a mythical idea of a rule – flying through the whole arithmetical series'.

He clarifies this point further by means of the notion of 'a machine as a symbol' (PI, 193). When we think of a machine as containing all its movements, we are not thinking of actual machines. For parts of an actual machine might bend, break off, melt and so on. In that case, one cannot even predict all its movements correctly. We can say that a machine completely determines all its movements only when we treat the machine as a symbol, and not as a physical mechanism. If we treat the machine symbolically (say e. g. representationally as it is evident from LFM (p. 195-196)) in order to make

[6] Diamond C. Wittgenstein's Lectures on the Foundations of Mathematics. – Sussex: Harvester Press, 1976, p. 29.

calculations, then the question of the representation going wrong does not arise. But a machine represented ideally i. e., symbolically, does not move whereas the machine as a physical mechanism can move, although the possibility of bending and breaking off in this case cannot be set aside. Wittgenstein insists that we conflate these two ideas when we form the notion of an ideal moving machine, which mysteriously predetermines all its movements accurately. Similarly, when we say that a formula e. g. $Y = X^2$ predetermines all the steps accurately; we are under the grip of the same misleading picture. We conflate the idea of a rule taken as a standard like the operation of mathematics, where mistake is inconceivable with the notion of a rule, which is involved in one's actual rule – following, where one can go wrong at any time. Thus we get the mythical notion of a rule, where its future applications are in a mysterious sense already present.

C: The Platonic account of necessity: The steps are determined by the discoverable mathematical and logical reality.

The Platonic picture of robust reality saves us from the disgrace of succumbing to Psychologism. G. H. Hardy, a famous mathematician, belongs to this group. He says:

> 317 is a prime not because we think so or because our minds are shaped in one way rather than another, but because it is so, because mathematical reality is built that way[7].

Gottlob Frege and Bertrand Russell – two of Wittgenstein's mentors – belonged to this group. To them, the problem of the student who says that he is following the rule 'add 2', but after 1000, continues the series like 1004, 1008, 1012, does not arise. Because, the platonist is confident that if the student says anything other than 1002 after 1000 , then he is simply going wrong; and if the stubborn pupil continues to go on in his way, and says that he is following the rule – then the Platonist will utter the Fregean phrase: 'Here we have a hitherto unknown kind of insanity'.

Wittgenstein is opposed to the idea of pure mathematics and logic as describing objective reality. Throughout his career, he struggled against this notion. In the Tractatus, his opposition to Platonism was obvious in his declaration that propositions of Logic and Mathematics are Sinnloss (i. e., without content), as they are not pictures of reality, and logical constants are not representations of reality.

[7] Hardy G. H. A Mathematician's Apology. – Cambridge: Cambridge University Press, 1967, p. 130.

Later Wittgenstein was more radical. He rejected this metaphysical picture of mathematical reality on the ground that it has got no connection with our ordinary life, language and behaviour. He says:

> What we call counting is an important part of our life's activities[8]
>
> and
>
> thinking and inferring like counting is of course bounded for us not by an arbitrary definition but by natural limits corresponding to the body of what can be called the role of thinking and inferring in our life[9].

D: The Sociological Account of necessity: The steps are determined by the conventions of our using language. It might be called sociological account of necessity.

In the history of philosophy, we find empiricists and logical positivists advocating this kind of necessity. In the transitional period, Wittgenstein came close to the thesis of logical positivism, which was reflected in his writings, posthumously published as Philosophical Remarks. But his later account of necessity differs widely from that of logical positivists and empiricists. In his later account of Philosophy of Mathematics, he clearly distinguishes mathematical propositions as such from empirical propositions in the sense that in an empirical experiment or calculation, different results are expected; while in Mathematics, there is only one possible result. He says in RFM:

> Within mathematics a proposition is not revisable by experience, mathematics as such is always measure, not things measured. (RFM, III-75, p. 201).

Such remarks vindicate the claim that he was not an empiricist as far as the notion of necessity is concerned, nor is he a logical positivist. In the RFM we find Wittgenstein stating categorically that propositions of mathematics and logic are synthetic – (RFM, II, 22, p. 132 and III, 42, p. 173), whereas the champions of logical positivism treat them as analytic.

The above discussions suggest that Wittgenstein opposes the psychological, functional, Platonist and sociological accounts of necessity. We now move over to the second section of this paper to find out the positive implications of his objections against the conventional thinking about

[8] Wittgenstein L. Remarks on the Foundations of Mathematics, p. 429.
[9] Ibid., p. 114.

necessity, and to find out (if any) an account of necessity in the writings of later Wittgenstein.

II

So far we have seen that for Wittgenstein, logical and mathematical propositions are different from empirical ones, and we apply the word "must" inexorably in mathematical, logical and grammatical propositions.

Wittgenstein clearly says:

> So much is clear: when someone says: "If you follow the rule, it must be like this", he has not any clear concept of what experience would correspond to the opposite.
>
> Or again: he has not any clear concept of what it would be like for it to be otherwise. And this is very important[10].

Hence if anyone infers anything as he likes, Wittgenstein would say, that is not inference proper. The example of the obstinate student also does not pose problems for Wittgenstein. For him, whatever the student is doing, he is not adding.

But there are other places where Wittgenstein has remarked: series in a different way, he is not following a rule.

> Why do I always speak of being compelled by a rule, why not of the fact that I can choose to follow it? For that is equally important.
>
> But I do not want to say, either that the rule compels me to act like this, but that it makes it possible for me to hold by it and let it compel me[11].

Again in mathematics and logic,

> It is not our finding the proposition self-evidently true, but our making the self-evidence count – that makes it into a mathematical proposition[12].

[10] Ibid., III, 29, p. 164.
[11] Diamond C. Wittgenstein's Lectures on the Foundations of Mathematics, p. 241.
[12] Wittgenstein L. Remarks on the Foundations of Mathematics, p. 114.

Here his intention is clear. He means that a rule compels us only if we go along with it, or in a sense agree to be compelled by it. Does it imply then that human agreement decides the truth/falsity of a proposition or a correct/incorrect way of following a rule?

Wittgenstein also says:

> The mathematical proposition is grounded on a technique. And also in the physical and psychological facts that make the technique possible[13].

> Logic belongs to the natural history of man. And that is not combinable with the hardness of logical "must"[14].

At this point we feel completely baffled. On the one hand, he says that he believes in the inexorability and objectivity of "must"; and on the other hand, he says that it is we who are inexorable in applying them. How is it possible? How can we solve the riddle? Or can we really treat Wittgenstein as being inconsistent here?

We can well imagine now that Wittgenstein's second group of remarks have provoked commentators like Michael Dummett to interpret him as a 'full blooded conventionalist' (1959)[15], or as a 'thoroughgoing internalist' (1999)[16]; and Paul Ernest to treat him as 'a social constructivist' (1999)[17].

To Ernest 'mathematics is a social construction, a cultural product, fallible like any other branch of knowledge and the justification of mathematical knowledge rests on its quasi-empirical basis': He attributes this view to Wittgenstein.

Dummett also attributes the view that 'the logical necessity of any statement is always the direct expression of a linguistic convention' to Wittgenstein.

Such characterizations does not seem to get along with or do justice to Wittgenstein's later philosophy in general, and his view of necessity in

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[13] Ibid., VII-I.
[14] Ibid., VI-49A.
[15] Dummett M. Wittgenstein's Philosophy of Mathematics // The Philosophical Review, Vol. 68, 1959.
[16] Dummett M. Wittgenstein on Necessity // Dummett M. The Seas of Language. –
[17] Ernest P. Social Constructivism as a Philosophy of Mathematics: Radical Constructivism Rehabilitated // Forthcoming.

particular. Wittgenstein himself anticipated such characterizations, and said in RFM:

> "So Wittgenstein, you seem to say there is no such thing as this proposition necessarily following from that" — should we say: Because we point out that whatever rule and axioms you give, you can still apply them in ever so many ways – that this in some way undermines mathematical necessity[18]?

It is a fact that Wittgenstein never undermined logical necessity in the sense that he never thought that mathematical propositions are empirical generalizations. Nor did he think that the student of PI (PI, 185) is following the rule when he continued in a bizarre way. And we have evidences also for saying so (RFM, III, 29, p. 164).

But the trouble with mature Wittgenstein is that he does not offer any theory of logical necessity, nor does he want to provide justification (as Ernest reported) for the necessity of mathematical and logical propositions. Rather, he wants to have a clear view of the words 'necessity' and 'necessary proposition', as they are used in our ordinary language. He looks into the way the word 'necessity' and 'necessary proposition' are used in language, life and the world; and finds out that such propositions are being treated specially on account of their incorrigibility. This special inexorability in using "must", "necessity" or "logically necessary" does not lie in its correspondence with eternal, mathematical, or logical objects. Rather, there are certain laws or statements which are suggested to us by the empirical world. They are special in the sense that we train our children and students to adopt rigorously, until they always or almost always get the correct result. Wittgenstein says:

> There correspond to our laws of logic – Very general facts of daily experience which suggest the laws that we adopt (RFM, I, 118, p. 82).

If we try to use different laws, draw anomalous conclusions, then we run into practical difficulties. He states clearly:

> If you draw different conclusions, you do indeed get into conflict e. g. with society, and also with other practical consequences (RFM, I, 116, p. 80, V, 46, pp. 298-299).

[18] Diamond C. Op. cit., p. 241.

Now as we drill and train rigorously to see errors in calculation in a certain way 'we are brought to the idea that logical and mathematical inferences are incorrigible, not to be questioned'. Hence necessity follows from looking at these propositions in a certain way.

What is important is that 'looking at propositions in a special way' is not peculiar to logical and mathematical propositions, it applies even in empirical propositions like 'I have two hands', 'Here is a tree' and 'My name is P. S.'.

Wittgenstein clarifies his position in On Certainty. There, he gives a list of propositions which stand fast for us, which constitute our frame of reference. These include mathematical propositions, logical propositions and some empirical propositions as well. He treats these propositions as examples of that agreement in judgement which is needed if language is to be a means of communication (PI, 242). They constitute the framework on which the workings of our language are based. There are important passages in On Certainty, which show that there are senses in which our frame of reference merges with the notion of form of life. These propositions are special in the sense that they are the 'hinge propositions' on which everything is based.

But the most important thing to be mentioned here is that for Wittgenstein, these propositions are not absolute or fixed for all possible worlds. They also change with the change of time, place and ideology. (OC # 96, # 97, # 98)[19].

What follows from the above discussion is that for Wittgenstein, the notion of necessity is not absolute. He gives hints that necessary propositions may become contingent and contingent ones may become necessary in course of time. That is, Wittgenstein challenges the way we have seen the problem so far, the way we have dichotomized the world into necessary and contingent, the way we have settled that necessity and contingency are binary opposites. He shows that there are necessities and contingencies in life, and also necessary propositions and contingent propositions in language; but at the same time, he also holds that it is our attitude towards these propositions that make some necessary and incorrigible, and some other contingent and fallible. And the most striking thing about this position is that Wittgenstein is not offering a theory or an account of necessary propositions in contrast with other accounts. On the contrary, he is persuading us to look at the usage of the word 'necessity' in our language and life, he is persuading us to take a certain view, 'a certain attitude' towards mathematical and logical necessity.

[19] See Wittgenstein L. On Certainty. – Oxford: Basil Blackwell, 1967.

As adopting an attitude cannot be equated with proposing an account or a theory, his news on necessity cannot be labeled as conventionalism or Platonism or intuitionism. He says clearly in 1939 Lectures:

> We might as well say that we need, not an intuition at each step, but a decision. Actually there is neither. You do not make a decision: you simply do a certain thing. It is a question of a certain practice (p. 237). Suppose that I tell you to multiply 418 by 563. Do you decide how to apply this rule for multiplication? No: you just multiply . . . It is not a decision (p. 238).

In fact if we attempt to label Wittgenstein's views as a theory, we'll be doing injustice to him. And while doing philosophy, philosophers interpret everything in terms of theories without paying attention to the ways the words are used in our ordinary life, and they thus get into trouble.

As regards the inexorability of logic, Wittgenstein thinks that the word 'inexorable' is used in a variety of ways in our ordinary language. Law of nature are inexorable. Laws of logic are inexorable. We also talk of inexorability in connection with people who punish. But if we adopt an attitude that inexorability belongs only to necessary propositions, then

> "We are like savages, primitive people, who hear the expressions of civilized men, put a false interpretation on them and draw queer conclusions from it[20].

Thus, as far as the notion of necessity is concerned, it would surely be wrong to accuse Wittgenstein of being a conventionalist or a social constructivist, or even of being a theorist at all.

[20] Wittgenstein L. Philosophical Investigations, p. 118.